Adaptive Channel Equalization in the Time-Varying Underwater Acoustic Channel: Performance Characterization and Robust Equalizers

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Abstract Channel-estimate-based equalizers are adaptive coherent equalizers for which observations of the received signal are used to estimate channel parameters, and these estimates are used to calculate the equalizer filter weights. Traditional channel-estimate-based equalizers calculate filter weights assuming that the estimates of the channel parameters are perfect. This work presents a common framework for evaluating both the performance of channel-estimate-based equalizers when the channel estimates are perfect (i.e., the minimal achievable error of the equalizer) and the degradation in performance of these equalizers due to errors in the channel estimates (i.e., the excess error).

For the three type of equalizers considered, (DFE, Linear MMSE, and Passive Time Reversal) the expressions for minimal achievable error take the form of the results from classical estimation theory for estimation error achieved by MMSE and matched filter estimators. These expressions are interpreted to give insights into the characteristics of "good" and "bad" channels. For the case when the channel estimates are MMSE estimates of the channel-impulse response, the excess error is shown to be proportional to the 2-norm of the calculated feedforward filter weight vector of the equalizer. This result is analogous to the "white noise gain" result characterizing the sensitivity of adaptive array processors to mismatch. This result is used to evaluate the relative sensitivity of all three types of equalizers to environmental mismatch. The analytic predictions of equalizer performance are compared with observed performance using data from several field experiments in different underwater acoustic environments.

The expressions for minimal achievable error and excess error give insights into potential methods of improving the robustness to channel mismatch of adaptive equalizers such as the DFE. Several of these methods are implemented and evaluated.

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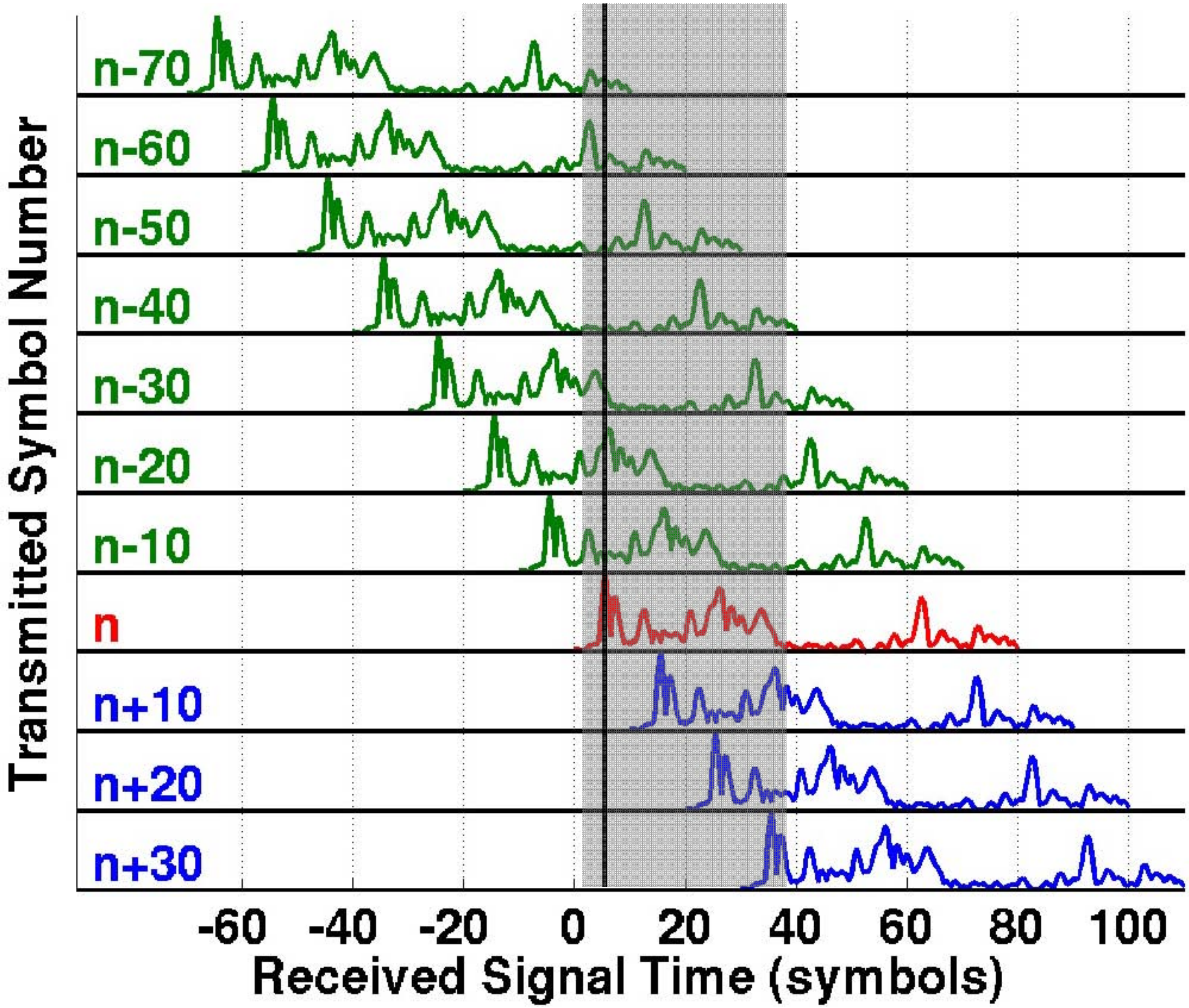
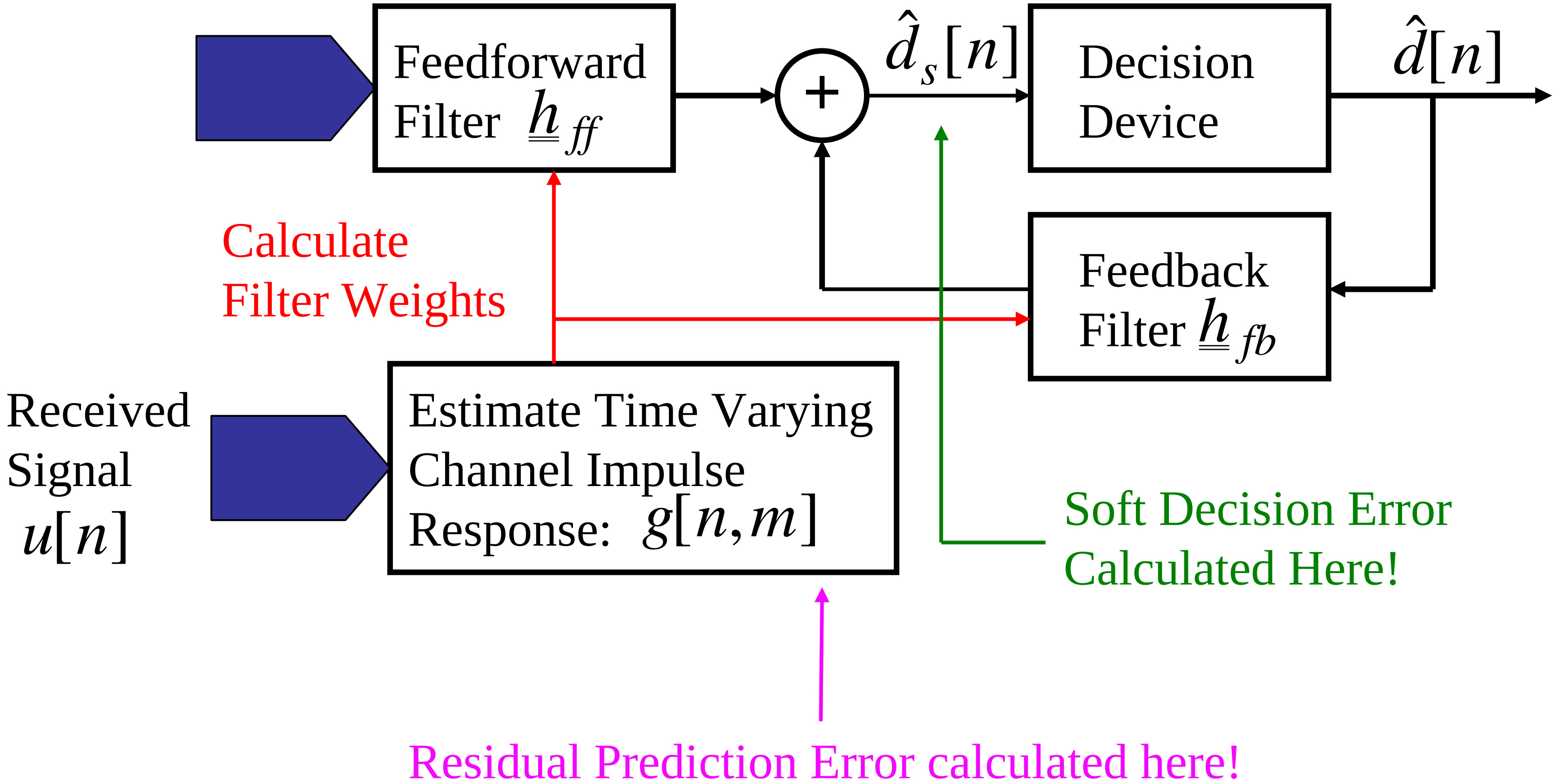
- Equalizers, Array Processing, and Performance Prediction
- SPACE02 Data: P-TR and DFE performance comparison.
- DFE Performance Analysis
- Robust Decision Feedback Equalization
- Conclusions

Intersymbol Interference and Channel Replica Vectors



Channel Estimate Based Equalizer Structures



Input to Feedforward Filter is a Time Series of Rec. Sig.

- $$\underline{\underline{u}}[n] = \begin{bmatrix} u[n-L_c] \\ \vdots \\ u[n] \\ \vdots \\ u[n+L_a] \end{bmatrix} \quad \underline{\underline{d}}[n] = \begin{bmatrix} d[n-L_c-N_c] \\ \vdots \\ d[n] \\ \vdots \\ d[n+L_a+N_a] \end{bmatrix} \quad \underline{\underline{v}}[n] = \begin{bmatrix} v[n-L_c] \\ \vdots \\ v[n] \\ \vdots \\ v[n+L_a] \end{bmatrix}$$

- $$\underline{\underline{u}}[n] = G^h[n]\underline{\underline{d}}[n] + \underline{\underline{v}}[n]$$

- $$G^h[n] = \left[\underline{\underline{r}}_{(L_c+N_c)} \quad \cdots \quad \underline{\underline{r}}_1 \quad \underline{\underline{r}}_0 \quad \underline{\underline{r}}_{-1} \quad \cdots \quad \underline{\underline{r}}_{-(L_a+N_a)}\right]$$

  (columns are replica vectors of the transmitted data signals as they appear in the feedforward filter input signal)

Partitioning the Time Varying Channel Impulse Response

received signal due to desired data signal

received signal due to data signals spanned by the feedback filter

received signal due to other data signals, “pre-cursor” replicas

- $$\underline{\underline{u}}[n] = \boxed{\underline{\underline{r}}_0[n]d[n]} + \boxed{G_{fb}^h[n]\underline{\underline{d}}_{fb}[n]} + \underbrace{\boxed{G_o^h[n]\underline{\underline{d}}_o[n]} + \underline{\underline{v}}[n]}_{\text{effective observation noise}}$$

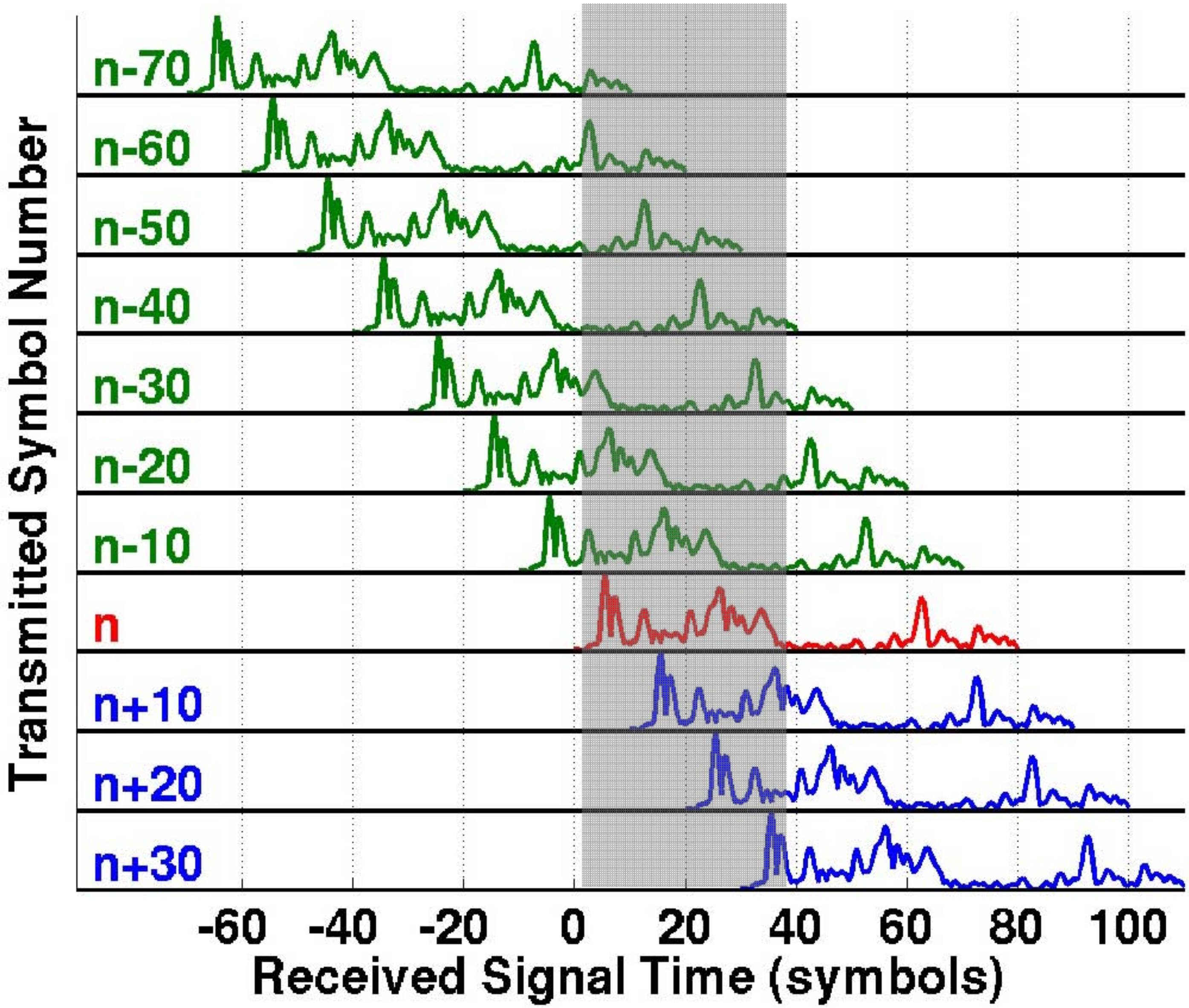
replica vector of desired signal

interference that can be subtracted from the received signal using the feedback filter

effective observation noise

- assume: zero mean, “white” data signal with energy $= \sigma_d^2 = 1$
  observation noise and data uncorrelated
- Normalized Effective Noise Correlation: $\boxed{Q = R_v + \hat{G}_o^h \hat{G}_o}$

Intersymbol Interference and Channel Replica Vectors



Coherent Equalizer Filter Weights

- Linear and DFE filter calculation formulated as a MMSE filtering problem:

$$\underline{\underline{h}}_{\text{opt}} = \arg\min \overline{\overline{\left| d - \hat{d}_s \right|^2}}$$

where $\overline{\overline{(\cdots)}}$ denotes expectation conditioned on the channel estimate

- Approach assumes that channel estimate has no error

- MMSE DFE: $\underline{\underline{h}}_{ff} = \dfrac{Q^{-1}\hat{\underline{\underline{r}}}_0}{1+\hat{\underline{\underline{r}}}_0^h Q^{-1}\hat{\underline{\underline{r}}}_0}$ $\underline{\underline{h}}_{fb} = -\hat{G}_{fb}\hat{\underline{\underline{h}}}_{ff}$

- MMSE Linear: $\underline{\underline{h}}_{lin} = \dfrac{(Q+\hat{G}_{fb}^h\hat{G}_{fb})^{-1}\hat{\underline{\underline{r}}}_0}{1+\hat{\underline{\underline{r}}}_0^h(Q+\hat{G}_{fb}^h\hat{G}_{fb})^{-1}\hat{\underline{\underline{r}}}_0}$

- Time Reversal: $\underline{\underline{h}}_{tr} = \dfrac{\hat{\underline{\underline{r}}}_0}{\hat{\underline{\underline{r}}}_0^h\hat{\underline{\underline{r}}}_0}$

Equalizer Performance with Perfect Channel Information

- Decompose Soft Decision Error into two components:

$$\sigma_s^2 = \overline{\overline{\left| d - \hat{d}_s \right|^2}} = \sigma_o^2 + \sigma_\varepsilon^2$$

- $$\sigma_{o_{dfe}}^2 = \frac{1}{1 + \underline{\underline{\hat{r}}}_0^h Q^{-1} \underline{\underline{\hat{r}}}_0} < \frac{\underline{\underline{\hat{r}}}_0^h Q \underline{\underline{\hat{r}}}_0}{\left(\underline{\underline{\hat{r}}}_0^h \underline{\underline{\hat{r}}}_0 \right)^2} < \frac{\underline{\underline{\hat{r}}}_0^h (Q + \hat{G}_{fb}^h \hat{G}_{fb}) \underline{\underline{\hat{r}}}_0}{\left(\underline{\underline{\hat{r}}}_0^h \underline{\underline{\hat{r}}}_0 \right)^2} = \sigma_{o_{tr}}^2$$

- $$Q = \frac{R_v}{\sigma_d^2} + \hat{G}_o^h \hat{G}_o$$

Equalizer Performance Degradation with Channel Estimation Errors

- $G = \hat{G} + \mathrm{E}_G$
- Assume $\hat{G}$ is a MMSE estimate $\Rightarrow \ \overline{\underline{\underline{\varepsilon}}}_0, \ \overline{\overline{\mathrm{E}}}_{fb}, \ \overline{\overline{\mathrm{E}}}_o = 0$
- $\sigma^2_{\varepsilon_{dfe}} = \underline{\underline{h}}^h_{ff} \left(\overline{\overline{\mathrm{E}^h_G \mathrm{E}_G}} \right) \underline{\underline{h}}_{ff}$
- $\sigma^2_{\varepsilon_{tr}} = \underline{\underline{h}}^h_{tr} \left(\overline{\overline{\mathrm{E}^h_G \mathrm{E}_G}} \right) \underline{\underline{h}}_{tr}$
- $\left(\overline{\overline{\mathrm{E}^h_G \mathrm{E}_G}} \right)$ denotes the expectation conditioned on the estimate of the channel impulse response.
- Magnitude Squared of FEEDFORWARD filter weight vector is an important determinant of sensitivity to channel estimation errors. (White noise gain result)

MMSE DFE and Passive Time Reversal Sensitivity Comparison

Large Adaptive Processing Gain ➡ $\underline{\underline{\hat{r}}}_0^h Q^{-1} \underline{\underline{\hat{r}}}_0 >> 1$

$$\left\| \underline{\underline{h}}_{ff} \right\|^2 \approx \left\| \frac{Q^{-1} \underline{\underline{\hat{r}}}_0}{\underline{\underline{\hat{r}}}_0^h Q^{-1} \underline{\underline{\hat{r}}}_0} \right\|^2 \geq \left\| \frac{\underline{\underline{\hat{r}}}_0}{\left(\underline{\underline{\hat{r}}}_0^h \underline{\underline{\hat{r}}}_0 \right)} \right\|^2 = \left\| \underline{\underline{h}}_{tr} \right\|^2$$

Prediction of Excess Error

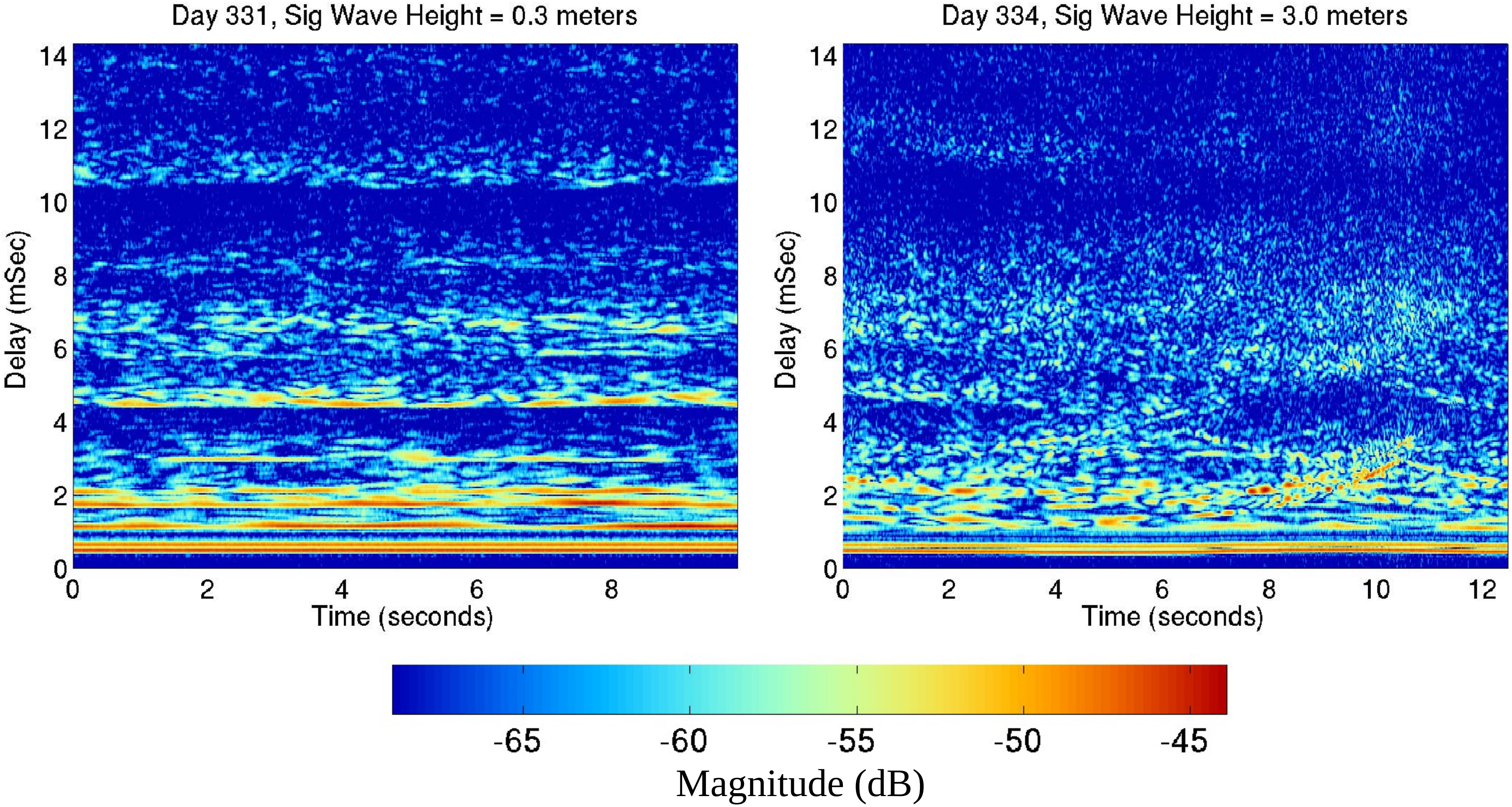
- $\sigma_{\varepsilon}^{2} = \underline{\underline{h}}_{ff}^{h} \left(\overline{\overline{\mathrm{E}_{G}^{h} \mathrm{E}_{G}}} \right) \underline{\underline{h}}_{ff}$
- $\mathrm{E}_{G} = G - \hat{G}$
- **Input signal to FF filter:** $\underline{\underline{u}}[n] = G^{h}[n] \underline{\underline{d}}[n] + \underline{\underline{v}}[n]$
- **Predicted Input signal to FF filter:** $\underline{\underline{\hat{u}}}[n] = \hat{G}^{h}[n] \underline{\underline{d}}[n]$
- **Residual Prediction Error:** $\underline{\underline{\varepsilon}}[n] = \underline{\underline{u}}[n] - \underline{\underline{\hat{u}}}[n] = E_{G}{}^{h}[n] \underline{\underline{d}}[n] + \underline{\underline{v}}[n]$
- $\sigma_{\varepsilon}^{2} \approx \underline{\underline{h}}_{ff}^{h} \left(\overline{\overline{\underline{\underline{\varepsilon}}\, \underline{\underline{\varepsilon}}^{h}}} \right) \underline{\underline{h}}_{ff}$

SPACE02 Experiment

- Multi-institution experiment (myself, Grant Deane (SIO), David Farmer (URI). Svein Vagle (IOS)). Fall 2002, 5 km off South coast of Martha's Vineyard, 15 meter water depth, benign topography.
- Data from portions of two days. Julian Date 331 when significant wave height was 0.3 meters, wind speed = 3 m/s. Julian Date 334 when significant wave height was 3.0 meters, wind speed building from 8.1 to 9.7 m/s.
- Data from one transmitter/receiver pair: Transmitter 6 meters above the bottom. 185 dB source level, 8 to 20 kHz bandwidth. 8 element vertical hydrophone array, 2 meter aperture, non-uniform spacing, bottom element 2 meters above the bottom. Horizontal range from source of 250 meters. (Fixed-fixed config.)
- 14 kHz carrier frequency. Binary phase shift keyed (bpsk) signals with symbol rate of 11161 symbols per second..

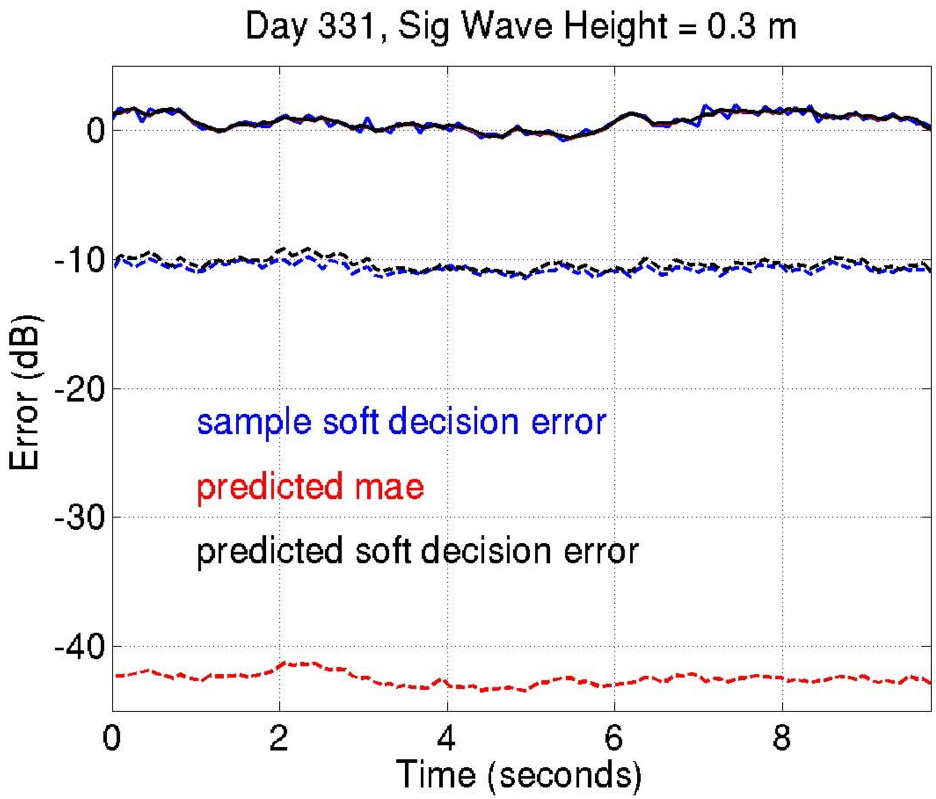
Estimates of Channel Impulse Response

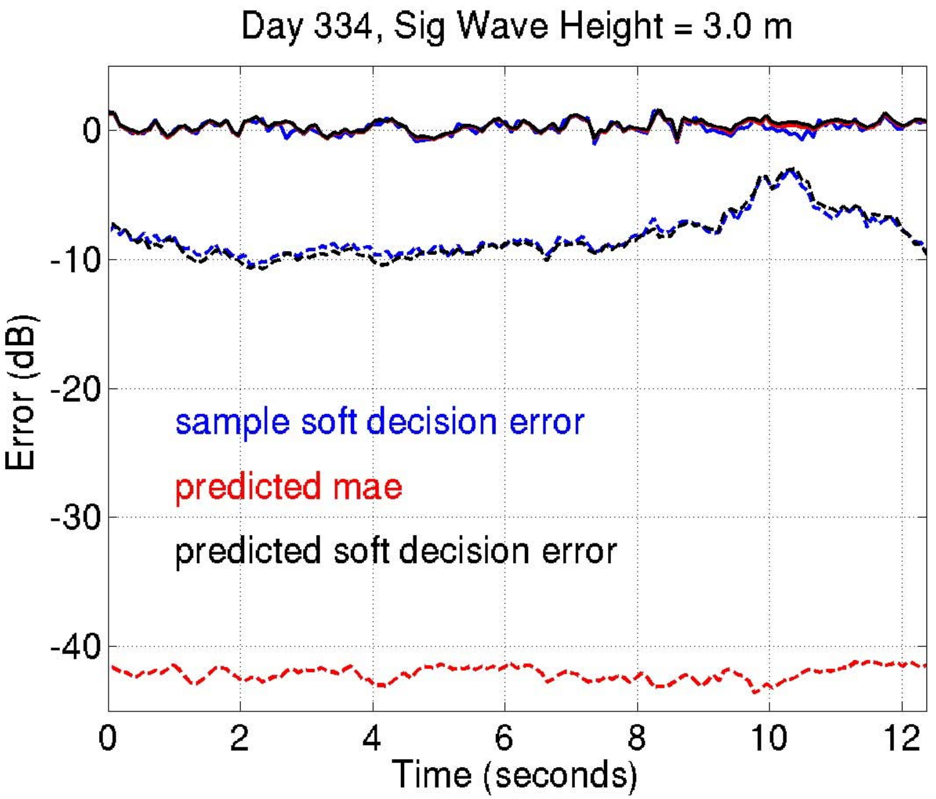
Least Squares Estimates using psk data (35.8 mSec rectangular averaging window)



- By most standards, these impulse responses would be considered to have a complex static structure.

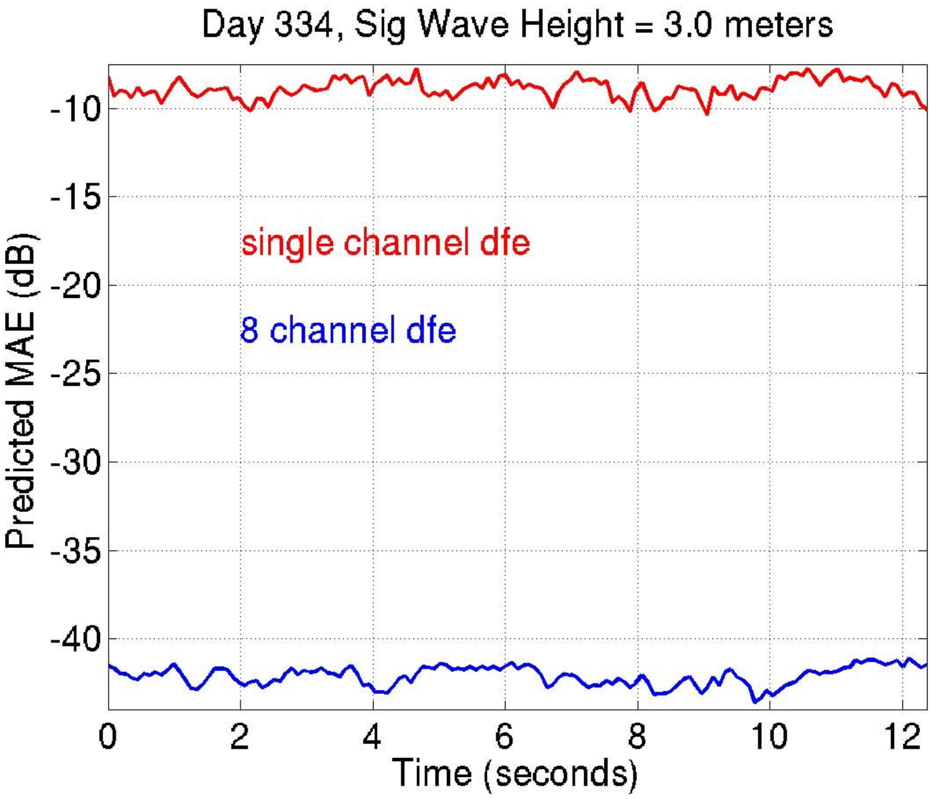
Eight Channel DFE and P-TR Equalizer Performance
Soft Decision Errors

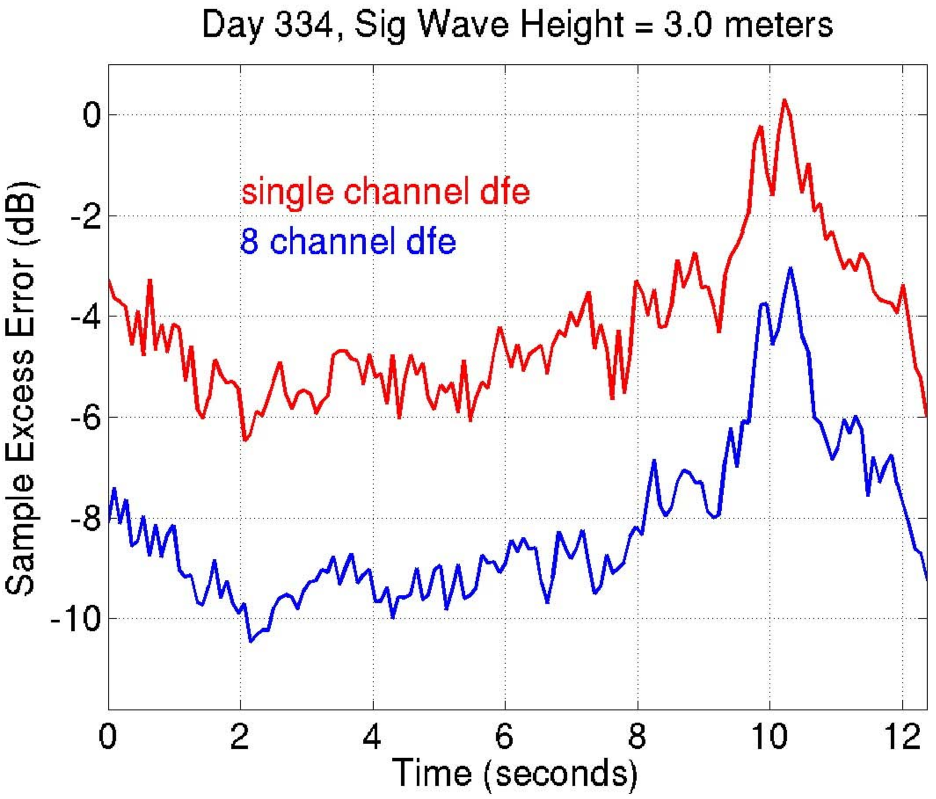




- For these complex channels, P-TR performance is limited by MAE (channel static structure) and is not significantly impacted by channel dynamics.
- For these complex channels, DFE performance is limited by the excess error caused by the dynamics and delay spread of the channel.

Single and Eight Channel DFE Performance Comparison





- Significant increase in single channel MAE.
- Single channel DFE error still dominated by excess error.
- Single channel DFE excess error greater than eight channel DFE excess error.

DFE Performance in Ocean Channels

- Even in channels with complex static structure, DFEs appear to be limited more by ability to track the channel than the ability to equalize the channel. (Excess error is limiting factor)

- Motivates development of equalizers that are robust with respect to channel estimation errors and improved techniques for tracking the time-varying channel impulse response.

Notes on Propagation Physics in Ocean Channels

- The taps that will contribute most to the channel tracking error are those that have high amplitudes and which fluctuate rapidly.
- Surface scattering is a primary source of high amplitude and rapidly fluctuating arrivals in channels with long delay spreads relative to the channel coherence time. (e.g., surface wave focused arrivals, Preisig and Deane, submitted to JASA)

The Development of Robust DFEs

- $\underline{\underline{u}}[n] = \hat{G}^h[n]\underline{\underline{d}}[n] + E_G{}^h[n]\underline{\underline{d}}[n] + \underline{\underline{v}}[n]$

Increase in the apparent observation noise level

- (Stojanovic, et. al., IEEE Trans. Comm., 1995) proposed increasing the assumed level of the observation noise by diagonal loading the assumed noise correlation matrix, $R_{\mathbf{v}}$

- Approach does not fully exploit the statistical structure of the apparent observation noise.

The Residual Prediction Error DFE

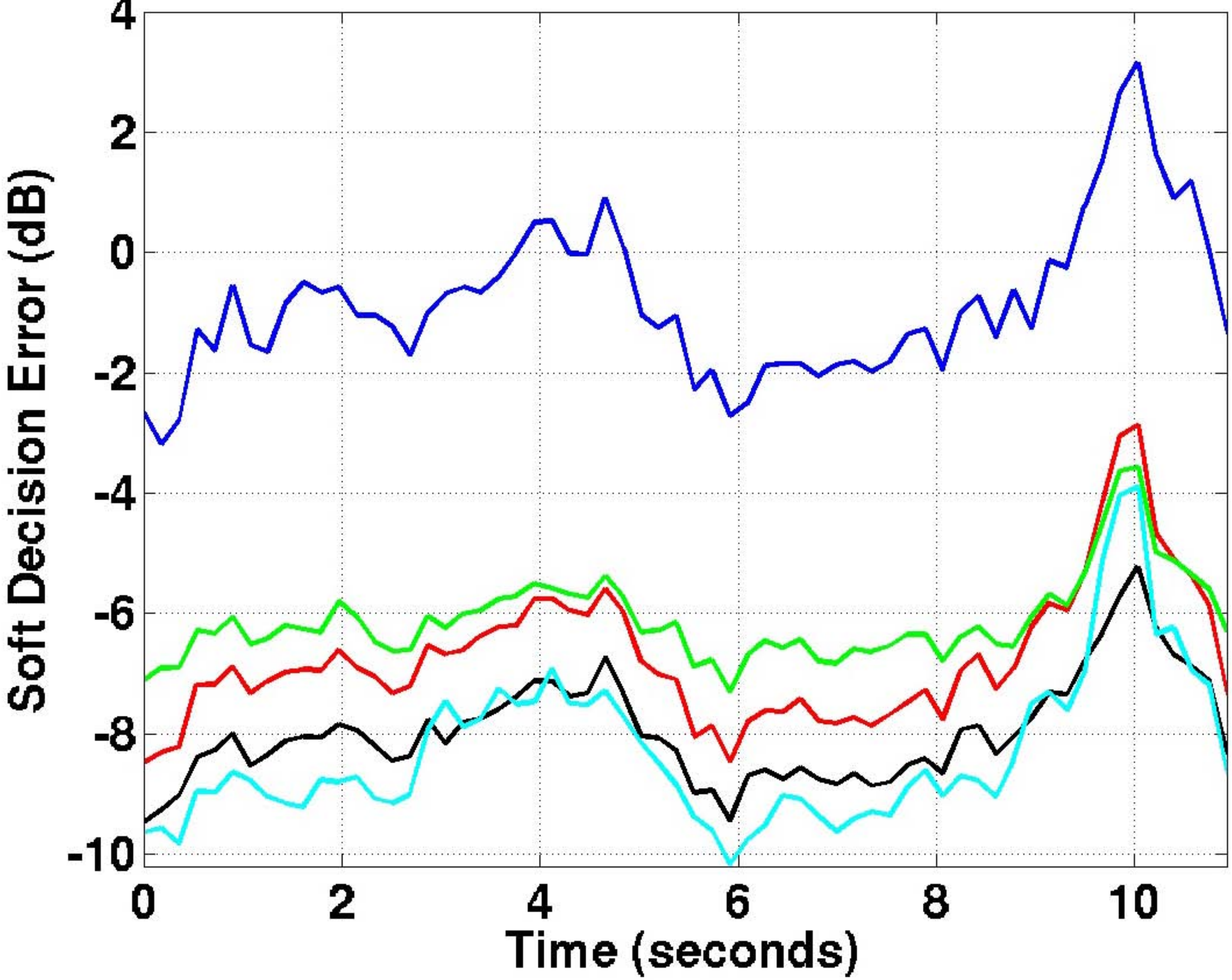
- $\underline{\underline{u}}[n] = \hat{G}^h[n]\underline{\underline{d}}[n] + \underline{\underline{\varepsilon}}[n]$

- $Q = R_\varepsilon + \hat{G}_o^h \hat{G}_o$

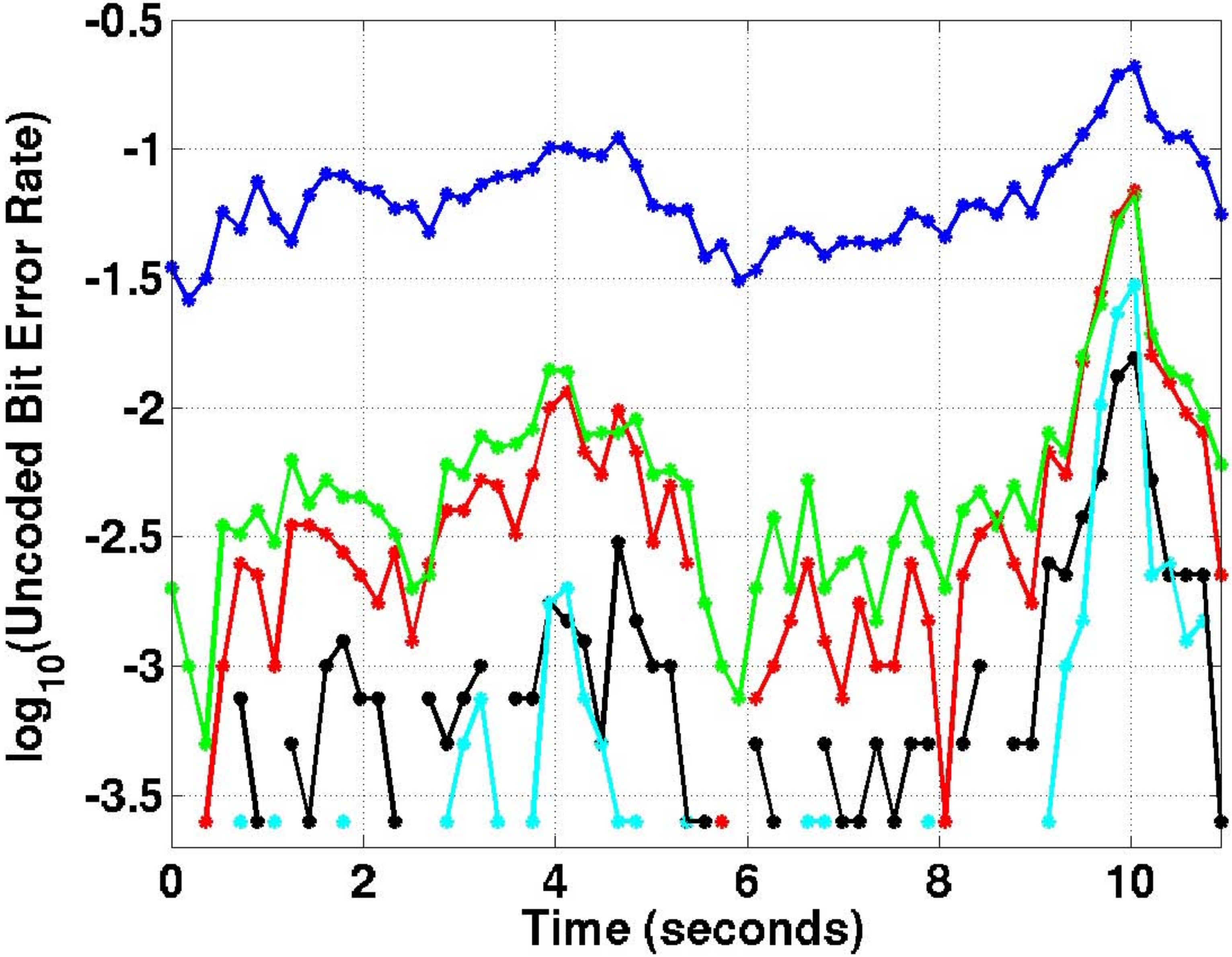
- Strict derivation of the “optimal” equalizer coefficients using the model in the first line above results in a Q matrix with other cross correlation matrices. However, these matrices cannot be estimated with sufficient accuracy to improve system performance.

- Empirical evidence suggests that the use of the additional terms results in a poorer system performance or equalizer failure.

DFE Performance Comparisons (SDE) for Day 334
3 and 8 channel equalizers



DFE Performance Comparisons (BER) for Day 334
3 and 8 channel equalizers



Summary of Equalizer Performance Analysis

- MMSE and passive time-reversal equalization can be put in a common framework and analytic expressions derived for their performance with and without channel estimation errors.
- The performance of these equalizers can be completely characterized by norm of the desired signal “replica vector” with respect to the “effective” noise correlation matrix.
- Sensitivity of processors to channel estimation errors depends on the magnitude squared of the feedforward or linear filter weight vector.
- MMSE DFE limited primarily by the ability to track channel fluctuations. The P-TR equalizer is limited primarily by the ability to equalize the channel.
- The modeling of the impact of channel estimation errors on the equalizer input signal as an increase in the observation noise level leads to the development of robust equalizers.
- The use of the residual prediction error of the equalizer input signal to calculate the statistics of the effective observation noise yields significant performance improvements.